Centripetal Motion and Gravity

Consider a satellite orbiting the Earth. It does so in a very nearly circular path, because:

- The spacecraft attempts to follow a "horizontal" straight-line path that is tangent to the curvature of the Earth.
- The Earth exerts a force of gravity on the satellite, pulling it downward much like an object that is thrown horizontally from a cliff.
- On a large scale, the Earth's surface is not flat; it curves away so that $\mathbf{F_g}$ always acts *radially*, i.e., into the center of the planet
- The balance between the inertia of the satellite attempting to go "straight" and the force of gravity $\mathbf{F_g}$ that pulls perpendicular to its motion causes the satellite to travel in its circular path around the planet, maintaining its elevation above Earth's surface. Note though:
  - if the satellite were to travel too fast, it would pull away from the Earth and no longer follow a circular path.
  - if the satellite were travelling too slow, it would also no longer follow a circular path; in this case, it would eventually fall into the Earth.

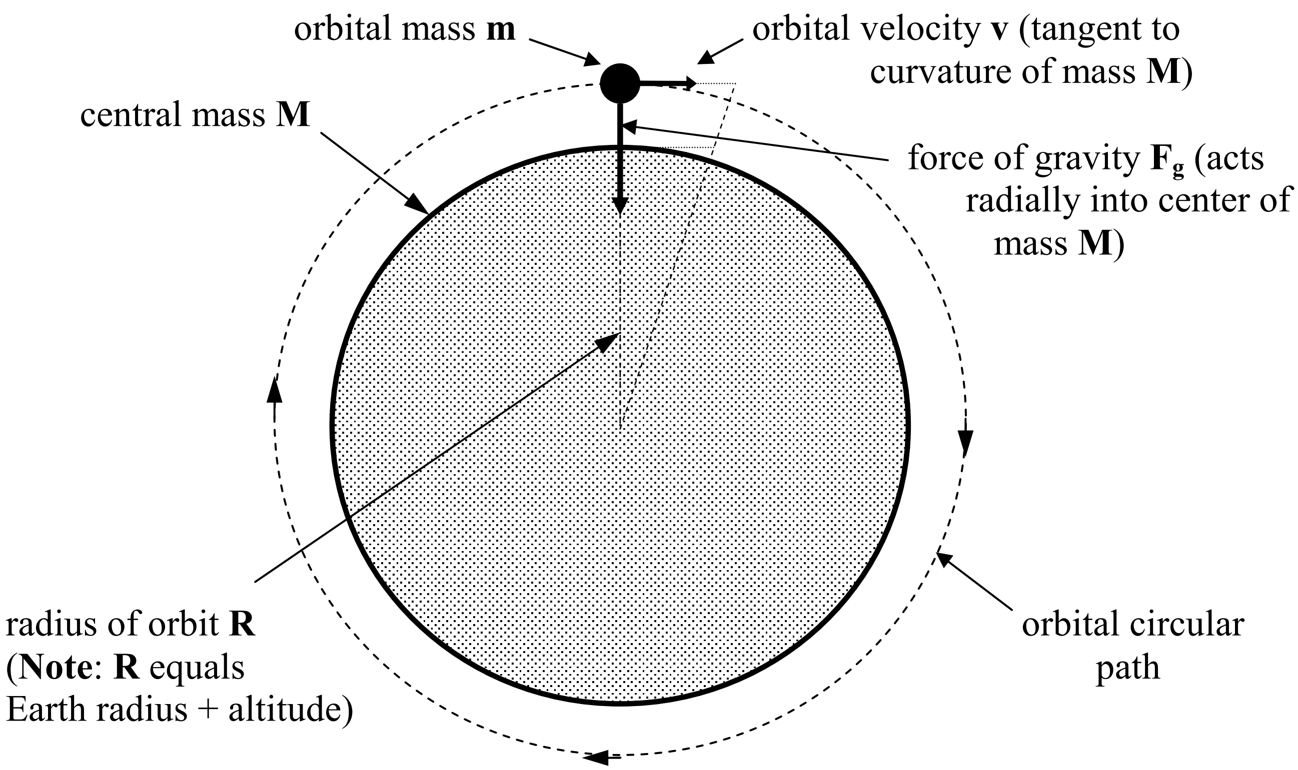
Stable, relatively circular orbits occur commonly in space. For example:

- natural satellites (called 'moons') orbit a central mass planet.
- planets orbit a central mass star (like the Sun).
- stars orbit the central matter of galaxies.

In each case, the *central mass* '**M**' supplies the force of gravity to maintain the circular orbit. The net force on the orbiting mass is a centripetal force, so we can state that, for any stable orbiting body:

$$\boxed{\mathbf{F_c = F_g}}$$

The next page illustrates the nature of a stable orbit.



The velocity for a stable satellite orbit can be determined in the following way:

- In this case, gravity provides the centripetal force; that is, $\mathbf{F_c = F_g}$
- Substitute in the formulas: $\mathbf{\frac{mv^2}{R} = \frac{GMm}{R^2}}$

  → cancel out orbital mass '**m**' to get $\mathbf{\frac{v^2}{R} = \frac{GM}{R^2}}$
- this tells us that $\mathbf{a_c = g}$ → the centripetal acceleration of the satellite must equal the gravitational field strength at that altitude!
- note that '**R**' also cancels once, so that $\mathbf{v = \sqrt{\frac{GM}{R}}}$

From these equations we see the following relationships:

- $\mathbf{F_g \alpha v^2}$
- $\mathbf{F_g \alpha 1/R}$
- $\mathbf{v \alpha \frac{1}{\sqrt{R}}}$

Example 6:

(a) **Determine the stable parking orbit velocity for a surveying satellite located 230 km above the moon's surface.**

(b) **If that orbital radius were reduced by one-tenth, by what factor would the orbiting speed increase?**

(see Gravitation Ex 6 for answer)